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The Rev. W. H. K. Soames, M.A., Chesterton Road, Cambridge;

were balloted for and duly elected Fellows of the Society.

Note giving a Numerical Illustration of the Effects of a Percentage Change on the Sun's Mean Motion in Longitude. By E. J. Stone, M.A., F.R.S.

The following is a direct numerical proof of the changes which must result from any alteration of the values of the sidereal times at mean noon adopted for the determination of the so-called mean solar times which are used for the comparison of any existing Lunar and Planetary Tables with observations.

I shall suppose a change of one per cent. to be made in the adopted value of the Sun's mean motion in longitude. This will render the effects due to the change distinctly visible in a few days, without affecting the generality of the reasoning. The effects due to the change made in 1864 can then be inferred directly by a simple proportion.

Now, taking the Greenwich mean noon of the *Nautical Almanac*, 1883 Dec. 1, as our epoch, and expressing our time,

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in terms of the adopted day we shall have, according to Le Verrier's Solar Tables

$$(1)\quad \text{Sun's mean longitude} = l = 249^\circ\ 59'\ 55''\cdot 22 + 3548''\cdot 330235\ .\ t,$$

from which we find by the usual formula for the Right Ascension of the Greenwich meridian

$$(2)\quad S = 16^h\ 39^m\ 59^s\cdot 68 + (24^h + 3^m\ 56^s\cdot 555349)t + \text{Nutation};$$

and let us assume that by the adoption of the value of l given in (1) we have found for the longitude of the Moon's centre of gravity

$$L = A + Nt,$$

and with the times t, deduced in the usual way from the adoption of the expression

$$S = 16^h\ 39^m\ 59^s\cdot 68 + 3^m\ 56^s\cdot 555349 \times t + \text{Nutation}$$

as the sidereal times at mean noon, we have found

$$A = 268^\circ \quad \text{and} \quad N = 47436''\cdot 72653,$$

so that

$$(3)\qquad L = 268^\circ + 47436''\cdot 72653 \times t.$$

But if we adopt for the Sun's mean longitude

$$(4)\qquad l = 249^\circ\ 59'\ 55''\cdot 22 + 3512''\cdot 84693268 \times t',$$

we should find from the usual formula for S

$$(5)\quad S' = 16^h\ 39^m\ 59^s\cdot 68 + (24^h + 3^m\ 54^s\cdot 18979551) \times t' + \text{Nutation},$$

and, although I will not assume it, from the necessary proportionality of the real motions of L, S, and l, however the units of time be changed, we should have

$$(6)\qquad L = 268^\circ + 46962''\cdot 3592647\ .\ t'.$$

The values of S and S′ at tabular mean noons, when $24^h \times t = 24^h \times t' = 24^h \times$ same integer, are given in the following table under the heads of A and B respectively.

Date.		Green. Mean Noon *N.A.* A h m s	Tabular Mean Noon. B h m s
1883, Dec.	1	16 40 0·18	16 40 0·18
	2	16 43 56·74	16 43 54·37
	3	16 47 53·29	16 47 48·56
	4	16 51 49·85	16 51 42·75
	5	16 55 46·41	16 55 36·95
	6	16 59 42·96	16 59 31·13
	7	17 3 39·52	17 3 25·33
	8	17 7 36·08	17 7 19·52
	9	17 11 32·63	17 11 13·71
	10	17 15 29·19	17 15 7·90
	11	17 19 25·75	17 19 2·09

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The results given under A are identical, as they should be, with those of the *Nautical Almanac.* Now, if we could assume, consistently with the conditions imposed upon us by the absolute necessity of (1), (2), and (3) representing the *same* real and independent motions before and after the change of the variable,

$$t = t'(1 - 0\cdot000027304334),$$

it would be found that S, which does equal

$$16^{\mathrm{h}}\ 39^{\mathrm{m}}\ 59^{\mathrm{s}}\cdot68 + (24^{\mathrm{h}} + 3^{\mathrm{m}}\ 56^{\mathrm{s}}\cdot555349)t + \text{Nutation}.$$

would become

$$16^{\mathrm{h}}\ 39^{\mathrm{m}}\ 59^{\mathrm{s}}\cdot68 + (24^{\mathrm{h}} + 3^{\mathrm{m}}\ 54^{\mathrm{s}}\cdot18979551)t' + \text{Nutation},$$

or S would be identical with S′, if the substitution is one consistent with the physical conditions of our problem.

It is upon the identity of S and S′ with this small change of t that all the opposition to the acceptance of my views is in reality based; and this is the supposed practical disproof of my results.

But if S does equal S′, then, as we are concerned with the mean longitude of the *real* Sun, and not with the longitude of an arbitrary mean Sun, we ought to have $l = l'$, which is not the case.

$$\begin{aligned} \text{For } l \text{ which} &= 249^\circ\ 59'\ 55''\cdot22 + 3548''\cdot330235 \times t \\ &= 249^\circ\ 59'\ 55''\cdot22 + 3548''\cdot330235 \,.\, t'(1 - 0\cdot000027304334) \\ &= 249^\circ\ 59'\ 55''\cdot22 + 3548''\cdot233350 \,.\, t. \end{aligned}$$

or expressed in time $= 16^{\mathrm{h}}\ 39^{\mathrm{m}}\ 59^{\mathrm{s}}\cdot68 + 3^{\mathrm{m}}\ 56^{\mathrm{s}}\cdot548890 \,.\, t'$, and consequently, therefore, we shall have

			h	m	s
Dec. 1.	R.A. of meridian at tabular mean noon	=	16	39	59·68
	Mean longitude of Sun in time at —ditto—	=	16	39	59·68
	Difference ...	=	0	0	0·00
Dec. 11.	R.A. of meridian at tabular mean noon	=	17	19	1·58
	Mean longitude of Sun at — ditto =	=	17	19	25·14
	Difference ...	=	0	0	23·56

and *as we have made the positions of the meridian agree whether we are using t times or t′ times by the relationship*

$$t = t'(1 - 0\cdot000027304334),$$

the adjustment of the Sun to the meridian cannot be made by shifting the position of the meridian, and must therefore be due to an erroneous allowance for the Sun's motion; a result which is

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directly indicated by the differences between the values of l and l'.

Conversely, if we attempt to transfer the expressions (4) and (5) by assuming that

$$t' = t(1 + 0{\cdot}0000273),$$

into the expressions (1) and (2), it will be found that although the values of S will agree, the values of the Sun's mean longitude will disagree; and the same thing will be brought out by appealing to the longitudes of the Moon.

I have already given in my papers the explanation of these difficulties. They are due to the neglect of the unit of time T in the equation of condition by which we have eliminated ω from our theoretical investigations, on the assumption that

$$\omega = 360 \times 60 \times 60'' + n, \text{ instead of } \frac{360 \times 60 \times 60}{\mathrm{T}} + n.$$

The existence of the factor T leads at once to the necessity of a correction $-0^{\mathrm{h}}{\cdot}24 \times t'$, before the results of Table B can replace those of Table A.

I am afraid that Prof. Adams has hardly sufficiently considered the fact that although the rotation of the Earth rapidly destroys angular errors and thus leads to small corrections of time when we determine the time directly from observations; yet these small corrections are not effective in removing the discordances between the tabular places and the observations. The tabular R.A. of the Moon from Hansen's Tables are now nearly a second of time too large. If we were to determine the clock errors from the tabular R.A. of the Moon instead of from our sidereal observations, we should only, therefore, deduce a correction of about a second in time to the adopted G.M.T. But this correction of time would not destroy the discordance between the tabular place of the Moon and observation. It is impossible to play fast and loose with the meridian in this way. The rapid motion of the Earth is as effective in displacing the relative positions of the meridian and any object in a given time δt as it is effective in destroying a given angular error in the time δt.

With Prof. Cayley's views I quite agree, except that, as a matter of fact, we do not at present make any distinction between the real Sun's mean longitude and that of the longitude of the mean Sun; and that we cannot therefore admit such distinctions in practice. If we did it would lead to my results.

For when we adopted with my notation α_1, we should have $n_1 = \alpha_1 + x_1$, and when we adopted α_2 we should have $n_2 = \alpha_2 + x_2$, and the relation between x_1 and x_2 would not be $\alpha_2 - \alpha_1$, since I have shown it to be impossible that $n_2 - n_1$ should equal $\alpha_2 - \alpha_1$, when we are dealing with the Sun's *real* motion in longitude. We cannot measure motion accurately in mean solar time so long as mean solar time is itself variable. And the corrections required for the breaks of continuity must be consistent with the founda-

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tions of our theoretical investigations, that in each unit of time the Sun's mean longitude did increase some definite and assigned angle, or these breaks of continuity will destroy all existing tables and introduce apparent inequalities not due to gravitation.

Note on Professor Adams's paper in the Monthly Notices for December 1883. By E. J. Stone, M.A., F.R.S.

Professor Adams's argument respecting the small difference between Bessel's and Le Verrier's sidereal times at mean noon is sufficiently treated on pages 36 and 37. It is impossible to assume, as Professor Adams does, that the meridian can have two directions in space at the same instant of absolute time.

Professor Adams's argument, page 46, line 11, that "mean solar time is measured, not by the Sun's mean motion in longitude, as Mr. Stone's theory supposes, but by the motion of the Sun in hour angle," is one which I do not profess to understand. However the mean solar time is determined, the tabular motion in longitude must represent the true motion of the Sun in longitude, if the tables are to agree with observations. That I, therefore, do not allow an arbitrary change in the adopted values of the sidereal times at mean noon to change the tabular motion in longitude, whilst Prof. Adams confessedly allows such changes to be introduced, appears to prove that I am right and Prof. Adams is wrong.

On page 46 Professor Adams proves correctly enough that with my α_1 and α_2 he has—

$$\frac{T_1-T_2}{T_2}=\frac{\alpha_1-\alpha_2}{360^\circ+\alpha_2}, \quad \text{or} \quad =\frac{\delta n}{360^\circ+n}$$

with Professor Adams's notation, but it is impossible that both α_1 and α_2 or n and $n+\delta n$ can represent the real motion of the Sun in longitude in the units T_1 and T_2. (See page 34.) My result on this point is confirmed by Professor Cayley, who gets my result if the expression for the mean longitude of the sun is any continuous function. (See page 48.)

It is true that mean time can be determined from the time of the Sun's transit and the equation of time. The error in time thus determined will not, however, remove the errors of the tables in longitude. It merely expresses the time which the meridian and not the Sun takes in moving from the tabular position to the true position, and such corrections can therefore only remove the $\frac{1}{366}$th part of tabular errors due to arbitrary changes in the adopted R.A. of meridian at mean noons. Take for example:—

On 1881 September 16 the observed R.A. of the Sun on the Greenwich meridian, with error of sidereal clock found from stellar observations, was $11^h\ 40^m\ 33^s{\cdot}05$, whilst the tabular R.A.